# Positive Definite and Semi-Definite Tensors

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## Abstract

Positive definite matrices and positive semi-definite matrices are very important concepts of matrix analysis in both theory and applications. In the recent decade, these two concepts have been extended to tensors. Research on positive definite tensors and positive semi-definite tensors has made significant progresses. In this talk, I will review these progresses. The main contents are as follows.

Except the zero tensor, there are no odd order positive semi-definite tensors. An even order symmetric tensor is positive semi-definite (definite) if and only if its smallest H-eigenvalue or $Z$-eigenvalue is nonnegative (positive). Even order diagonally dominated symmetric tensors, even order symmetric $B_0$ tensors, and even order Cauchy tensors with positive generating vectors are positive semi-definite tensors. They are easily checkable. Even order symmetric $M$ tensors, even order completely positive tensors, even order Laplacian tensors, even order signless Laplacian tensors, even order strong Hankel tensors, even order complete Hankel tensors, even order Hilbert tensors, even order symmetric $P_0$ tensors, even order diagonally dominated circulant tensors, even order circulant $B_0$ tensors, the Motzkin tensor are all positive semi-definite tensors. We now have a rich theory on positive definite and semi-definite tensors.