Some Selected Applications of Certain Minimax Theorems

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Abstract

In this lecture, I intend to present a selection of the various applications of certain minimax theorems. In particular, I will consider operators $T : X \to X$ of the type $T = \mathrm{id} + J'$ where X is an infinite-dimensional real Hilbert space and J' is the Gâteaux derivative of a C^1 functional $J : X \to \mathbf{R}$. For such operators the following properties will be highlighted. If J is sequentially weakly lower semicontinuous and J' is non-expansive, then there exists a closed ball B in X such that $T(B)$ intersects every convex and dense subset of X. If J' is compact, $\liminf_{\|x\|\to+\infty} \|x\|^{-2} J(x) > -2^{-1}$, T is non-monotone and $\lim_{\|x\|\to+\infty} \|T(x)\| = +\infty$, then the set of all singular points of T is not σ-compact.